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DEVELOPMENT OF THE CONTROL THEORIES AND METHODS FOR  
FOR OPTIMAL RENDEZVOUS IN SPACE



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ABSTRACT This article describes in a general way optimal rendezvous control theories and methods inside and outside China. It stresses a discussion of progress in China in the last few years. It introduces the results of research on the theories and methods of adjacent near circular orbit optimal rendezvous under the effects of finite thrusts, ordinary coplanar elliptical orbit general optimal rendezvous, multiple pulse thrust linearized and nonlinear rendezvous orbital optimal guidance and horizontal impulse thrust optimal rendezvous control.

SUBJECT TERMS Rendezvous Docking Optimal rendezvous Control theory

## I. INTRODUCTION

Space rendezvous and docking is an important high technology of space navigation. It is a prerequisite condition for such high level space operations as the assembly, recovery, resupply, and repair and maintenance as well as international space rescue vessel support missions to realize such fundamental, large model space facilities as space stations, space laboratories, space communications and remote sensing platforms, and so on, in orbit. Future space missions--for example, space materials working and near earth orbit platforms used for observations of the earth and the heavens as well as geosynchronous orbital platforms for long range communications missions--all require the use of rendezvous and docking technology. For instance, the U.S. space shuttle will carry out orbital repair and maintenance on the Hubble telescope.

Space rendezvous and docking refers to two space craft in orbit rendezvousing with each other in accordance with predetermined times, using the same speeds, at predetermined positions in space, and, in conjunction with this, through docking mechanisms, the entire process of connecting structurally to form one single whole. As far as the two space craft are concerned, one is passive. It moves in a predetermined orbit. It is called the target flight craft. The other space craft is active. It is called the tracking flight craft. It completes all the orbital maneuver

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\* Numbers in margins indicate foreign pagination.  
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missions required for rendezvous and docking--finally, rendezvousing with the target flight craft and, in conjunction with that, docking together.

Due to the fact that the propellants carried by space craft are always limited, during the process of space craft changing orbits, if fuel consumption is reduced as much as possible, useful loads of space craft can be increased and economic benefits raised. As a result, such problems as space craft changing orbits with as little fuel as possible and rendezvousing with as little fuel as possible are always questions of very great interest to theoretical workers [1].

Although the U.S. and the Soviets have already realized rendezvous and docking, according to the introductions of relevant experts, however, the rendezvous and docking, fuel costs, time period lengths, success rates, as well as autonomous capabilities and precision all await improvements. At the present time, they are in the process of researching questions associated with high precision rendezvous guidance, navigation, and control with as little fuel as possible.

## II. THEORY OF OPTIMAL RENDEZVOUS ASSOCIATED WITH FINITE THRUST EFFECTS

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In practical engineering, engine thrusts are all finite. As a result, as far as research with the effects of finite thrusts is concerned, the rendezvous problem associated with two flight craft is a subject brought up by the realities of engineering.

Articles researching problems of flight craft rendezvous under the effects of finite thrusts, up to the present time, are

still relatively few. A number of scholars in China have carried out research on this problem [2-5]. Reference [2] studies rendezvous problems associated with two flight craft in adjacent near circular orbits. Relative motion equations associated with the flight craft make use of C-W equations for descriptions. This article assumes that thrust accelerations can continue to change with time. Maximum value principles are used to solve for optimum thrust accelerations. Reference [3], besides studying impulse intercept and rendezvous problems, also researches finite thrust intercept questions. Reference [4] studies finite thrust rendezvous problems. Reference [5] studies near distance rendezvous problems. On the basis of C-W equations for the relative motions of two flight craft, it puts forward linear rendezvous distance-speed control algorithms. References [3] and [4] both opt for the use of space inertial coordinate systems and give relative motion equations for two flight craft in this coordinate system. After that, Pontriagin maximum value principles are used to carry out analysis and give the forms for optimum rendezvous guidance rules.

Scholars abroad have also only done some research on finite thrust rendezvous problems [6-11] in the last few years. Reference [6] studies optimal finite thrust rendezvous in near circular orbits. It uses C-W equations to describe the relative motions of two spacecraft. Taking the Lawden [12] necessary conditions relating to Primer vectors and substituting into equations, one obtains two point lateral value problems relating to optimal solutions. Reference [7] takes the results from reference [6] and expands them to a general Keplerian orbit. After taking spacecraft relative equations and linearizing them, Primer vector theory is used in the same way to obtain optimum solution conditions. References [6] and [7] both assume, during rendezvous processes, that the mass of the tracking flight craft is a constant. Reference [8] analyzes influences on rendezvous of reductions in the mass of tracking flight craft due to fuel consumption. Reference [9], through an integration expression

revised by citations from Lawden [12] relating to "Primer vectors", obtains a new form for rendezvous equations in order to shift out points of singularity. Reference [10] takes the results from references [6-9] and generalizes them to an ordinary central gravity field. With regard to the status of this type of gravitational field, relative motion equations for flight craft are derived and rendezvous problems studied.

Reference [11] uses nonlinear program methods to research flight craft orbits associated with optimal finite thrust effects. It takes flight craft orbits and divides them into certain sections. It takes status vectors for various nodal points, control vectors, and areas of thrust arc to act as variables and sums of thrust arc zones to act as target functions. From such things as status equations as well as initial and final conditions, and so on, it derives a number of limit conditions, finally reaching large scale nonlinear program problems.

In summary, with regard to optimal rendezvous problems under finite thrust effects, although some research has been done, this problem still, however, has no very good solution. The articles above, during their studies, assume that thrusts can be continuously changed. This is very difficult to realize in engineering terms. Although there are a number of articles which have considered rendezvous problems under the effects of normal thrusts which come the closest to engineering reality, if one wants to solve this problem, however, there are still very large difficulties existing.

In article [1], we studied problems of rendezvous under the effects of normal thrusts for two spacecraft close to each other in near circular orbits for fixed time periods and optimum fuel. On the basis of linearized C-W equations, use was made of maximum value principles to derive optimum thrust effect switching functions and thrust direction expressions. In conjunction with

this, research was carried out on orbital arc sections with thrust effects, respectively giving a number of properties of intermediate arc sections having thrust effects and final arc sections having thrust effects.

### III. OPTIMAL RENDEZVOUS THEORY FOR IMPULSE THRUST EFFECTS

The theory of impulse orbital changes has important significance for the study of problems associated with all of spacecraft rendezvous, interception, and orbital shifting. Due to the fact that it is relatively complicated to study these orbital maneuver problems under the effects of normal thrusts, and, moreover, when option is made to use large thrust motors to act as orbital maneuver system propulsion equipment, space flight craft can, within a short time, obtain the needed velocity increments of increase, as a result, during preliminary discussions of orbital maneuver problems, it is often assumed that engines operate in an impulse mode. Impulse thrust assumptions are as follows:

- 1) maximum thrusts of propulsion motors are boundless;
- 2) when maximum thrust effect times are extremely short, it is possible to believe that they are instantaneously completed;
- 3) within extremely short maximum thrust effect times, flight craft position changes can be ignored;
- 4) within extremely short maximum thrust effect time periods, flight craft velocity vectors give rise to finite changes, that is, at this instant, velocity vector alterations

With regard to problems of optimal rendezvous between two spacecraft under the effects of impulse thrusts, methods which scholars have made wide use of in their studies are Lawden's [12] "Primer vector" methods. The theory of "Primer vectors" which Lawden puts forward is appropriate for use on optimal flight path problems in inverse square law gravitational fields. It gives the necessary conditions for optimal flight orbits under impulse effects. Later, Lion and Handelman [13] take the concepts of "Primer vectors" and generalize them to reach nonoptimal flight orbits, establishing the necessary conditions associated with increasing intermediate impulses. Reference [14] applies the results of Lion and Handelman, giving methods to calculate optimal impulse orbits.

Below, two types of cases are distinguished and the general status of the development of optimal impulse rendezvous theory is introduced.

#### (I) Optimal Impulse Rendezvous Problems Based on Linearized Relative Motion Equations

In the earth's gravitational field--no matter whether it is motion equations for one flight craft or relative motion equations for two flight craft--in all cases, they are nonlinear. With this type of equation, studying flight craft rendezvous problems is relatively difficult. Moreover, when two flight craft are moving in two adjacent, near circular orbits and the distances between flight craft are relatively small (compared to the radius vector of the spacecraft), the relative motion equations for the two flight craft can be linearized. As a result, many scholars all study optimal impulse rendezvous problems based on linearized relative motion equations for the

two flight craft.

In the late 1960's, Prussing [15,16] applied Lawden's "Primer vector" theory to research fixed time impulse rendezvous problems between adjacent, coplanar circular orbits. He selected a movement point on a circular reference orbit to act as reference point. The radius of the reference orbit was fixed as the intermediate value of the orbital radii of target flight craft and tracking flight craft. With regard to the given number of impulses, Prussing gave a "Primer vector" solution satisfying Lawden's necessary conditions. Reference [15] discusses 4 impulse time fixed rendezvous problems between adjacent, coplanar circular orbits. Reference [16] discusses 2 impulse and 3 impulse time fixed rendezvous problems between adjacent, coplanar circular orbits. Although Prussing's methods are simple and convenient, the results he obtains, however, are not very ideal. Velocity increments of increase associated with orbital transformations are very sensitive to position errors in impulse points. Liu and Plexico [17] modified Prussing's methods. They cited a reference orbit associated with a minimized impulse point angular position. Reference points moving in reference orbits possess angular velocities intermediate in the process of tracking flight craft completing entire orbital transformations. Their results were more accurate than Prussing's results, reducing velocity increment of increase sensitivity to impulse point position errors. However, among the optimization solutions they obtained, when velocity increments of increase grow larger than included horizon angles at locations in question--in particular, when tracking flight craft orbit eccentricities are not zero--the offset angles become very large.

In 1960, Chohessy and Wiltshire [18] derived, in a target orbit coordinate system, linearized relative motion equations associated with two spacecraft, that is, the famous C-W equations. C-W equations are widely used in studying rendezvous

problems associated with two flight craft in adjacent near circular orbits. References [19] and [20] opt for the use of C-W equations to research problems of flight craft rendezvous associated with safety.

Chinese scholars--with regard to impulse rendezvous problems in linearized equations--have also done in depth research. Reference [21], based on C-W equations, studies problems of rendezvous between two flight craft with maximum fuel savings under the effects of 2 impulses, and, in conjunction with this, achieves understanding and analysis. Reference [22] researches problems of rendezvous between two flight craft with maximum fuel savings under the effects of 4 impulses, giving numerical value solutions for rendezvous problems satisfying Lawden's necessary conditions.

On the foundation of the previous people, we also relatively comprehensively studied problems of fixed time, maximum fuel saving rendezvous between two flight craft in adjacent near circular orbits under the effects of multiple impulse thrusts. On the basis of linearized C-W equations for relative flight craft motions, we put forward a type of method to solve problems associated with impulse rendezvous and maximum fuel savings using Lagrange factor methods. Using Lagrange factor methods, it is possible to calculate the magnitude and direction of rendezvous thrusts. In comparison to traditional Lawden methods, this type of method possesses the characteristics of simple derivations and convenient calculations. Moreover, with regard to cases with different numbers of impulses, it possesses universality. For actual methods, see Reference [23]. In Reference [23], we also give change curves for fuel consumption following along with rendezvous times, change curves for fuel consumption following along with initial conditions, and so on, and so on. These studies have very great significance in engineering terms.

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## (II) Optimal Impulse Rendezvous Problems Based on Nonlinear Motion Equations

Making use of linearized equations to study rendezvous problems is simple and convenient. With regard to rendezvous problems associated with flight craft in motion in two adjacent circular orbits, it is possible to supply relatively good approximations. However, in regard to actual problems, the range of suitable uses for this type of linearized treatment is relatively narrow. Even if flight craft rendezvous in two circular orbits, it is also only when there are very small distances between two spacecraft that use is then appropriate in the automatic homing stage of rendezvous and docking. Moreover, in the realities of engineering, in order to complete rendezvous and docking, making target orbits circular beforehand is not only not necessary, it is sometimes also not possible--for example, using the space shuttle to salvage a malfunctioning satellite. As a result, flight craft rendezvous control must make further use of nonlinear equations for research. The Jet Propulsion Laboratory subordinate to the U.S. N.A.S.A is also in the process of searching out a type of new guidance technology capable of being suitable to use in circular, elliptical, parabolic, and hyperbolic orbits as well as its algorithms. However, due to the complexity of nonlinear equations, references studying optimal rendezvous problems based on nonlinear equations are certainly not numerous.

Gross and Prussing [24] use "Primer vector" theory to research fixed time rendezvous problems associated with maximum fuel savings, 2 impulses and 3 impulses, and direct ascent. In Reference [25], Eckel discusses multiple impulse orbit shift problems associated with non coplanar elliptical orbits. He makes use of "Primer vector" theory, taking Lawden's necessary conditions to solve for optimal solutions and substituting into flight craft movement equations. After that, use is made of

maximum value principles to derive optimal solutions required to satisfy equations. However, how to obtain final results still awaits further discussion. Reference [26] applies general extreme value theory, discussing time fixed 2 impulse orbital shift problems between non coplanar elliptical orbits, and verifying that optimal solutions depend on the selection of three variables. They are transitional orbit semi parameters as well as two impulse point location true anomalies defined in initial and final orbits. However, how to solve for optimal solutions still needs more study. Chiu and Prussing [27] apply the methods of Reference [13], studying fixed time optimal rendezvous problems between two elliptical orbits. They respectively consider coplanar circular orbits and bound non coplanar circular orbit cases. When considering coplanar circular orbit rendezvous, assume that the initial points are on lines of intersection associated with the planes of two circular orbits. In articles, they give curves associated with fuel consumption following along with changes in rendezvous times. However, the methods they use are only capable of solving rendezvous problems of flight craft in two circular orbits. Moreover, in their articles, minimized algorithm convergence situations are certainly not good. It is very possible to converge to local optima. In summary, as far as the study of optimal flight craft rendezvous for general elliptical orbits on the basis of nonlinear equations is concerned, this problem has certainly not been resolved.

With regard to using nonlinear equations to study two flight craft general rendezvous problems in coplanar elliptical orbits, we put forward a type of "two layer dynamic program" method [28]. Calculations with this type of method are convenient. It is suitable for use on flight craft rendezvous problems in general coplanar elliptical orbits. Moreover, it guarantees fully optimal characteristics.

In actual engineering, the attitudes of a great many spacecraft are pointed toward the earth. Moreover, orbital change motors on flight craft are fixed to the main axis of the flight craft. As a result, in the directions of thrusts given in space, there must be a reliance on adjustments of flight craft attitude in order to guarantee it. Rendezvous in general will go through many iterations of orbital change controls. Each instance requires space craft attitudes to leave normal flight attitudes and turn in a certain direction. This will create very great pressures on flight craft inertial guidance systems and ground control. Also, it extends the rendezvous flight process. During the course of rendezvous, if there is a requirement for thrust direction to follow along the horizontal at a place in question, to a certain extent, this can resolve the problems discussed above. As a result, we studied flight craft orbital shift and orbital rendezvous problems [29,30] in coplanar elliptical orbits under the effects of horizontal impulses.

As far as adjacent, near circular orbit rendezvous line of sight guidance, also called parallel guidance, is concerned, its theory and methods were already successfully used, in the late 1960's, in rendezvous and docking of the Apollo space ships and Union space ships as well as the space laboratory.

In the 1970's, Chinese scholars had already done in depth research on space rendezvous optimal control theory. With the impetus and support of China's 863 high technology program, Chinese research on rendezvous and docking control theory and methods had a new development. In recent years, not only have multiple impulse thrust optimal rendezvous problems in adjacent near circular orbits been relatively well resolved, but, at the same time, research associated with elliptical orbit optimal rendezvous with finite normal thrusts has also made progress. In particular, in the case of the establishment of horizontal

impulse thrust optimal rendezvous theory and methods, there is relatively great practical value.

#### REFERENCES

- [1] 谢颖,“空间最优交会控制理论与方法研究”, 哈尔滨工业大学博士论文, 1992年6月。
- [2] 程国采,《弹道导弹制导方法与最优控制》, 国防科大出版社, 1987年9月。
- [3] 秦化淑, 王朝珠等,《大气层外拦截交会的导引问题》, 国防工业出版社, 1977年。
- [4] 王朝珠, 空间最优交会, 宇航学报, 1991年第4期。
- [5] 于绍华,“航天器自主交会运动轨迹的控制”, 宇航学报, 1993年第1期。
- [6] T.Carter, “Fuel–Optimal Maneuvers of A Spacecraft Relative to A Point Circular Orbit.” *Journal of Guidance, Control, and Dynamics*, Vol.7, No.6, 1984.
- [7] T.Carter, and M. Humi. “Fuel–Optimal Rendezvous Near A Point in General Keplerian Orbit.” *Journal of Guidance, Control, and Dynamics*, Vol. 10, No.6, 1987.
- [8] T.Carter, “Effects of Propellant Mass Loss on Fuel–Optimal Rendezvous Near Keplerian Orbit.” *Journal of Guidance, Control, and Dynamics*, Vol.12, No.1, 1989.
- [9] T.E.Carter. “New Form for the Optimal Rendezvous Equations Near A Keplerian Orbit.” *Journal of Guidance, Control, and Dynamics*, Vol.13, No.1, 1990.
- [10] Mayer Humi. “Fuel–Optimal Rendezvous in A General Central Force Field”, Vol.16, No.1, 1993.
- [11] P.J.Enright and B.A.Conway. Optimal Finite–Thrust Spacecraft Trajectories Using Collocation and Nonlinear Programming.” *Journal of Guidance, Control, and Dynamics*, Vol.14, No.5, 1991.
- [12] D.F. Lawden. *Optimal Trajectories for Space Navigation*. Butterworths, London, 1963.
- [13] P. M. Lion, and M. Handelman. “Primer Vector on Fixed–Time Impulsive Trajectories.” *AIAA Journal*, Vol.6, No. 1, Jan. 1969.
- [14] D.J.Jezewski, and H.L.Rozendaal. “An Efficient Method for Calculating Optimal Free–Space N–Impulse Trajectories.” *AIAA Journal*, Vol.6, No.11, Nov. 1968.
- [15] J.E.Prussing. “Optimal Four–Impulse Fixed–Time Rendezvous in the Vicinity of A Circular Orbit.” *AIAA Journal*, Vol. No. 5, May 1969.
- [16] J.E.Prussing. “Optimal Two and Three Impulse Fixed–Time Rendezvous in the Vicinity of A Circular Orbit.” *AIAA Journal*, Vol. 8, No.7, 1970.
- [17] F.C.Liu, and L.D.Piexico. “Improved Solution of Optimal Impulsive Fixed–Time Rendezvous.” *Journal of Spacecraft and Rocket*, Vol. 19, No. 6, 1982.
- [18] W.H.Clohessy, and R.S.Wiltshire, “Terminal Guidance System for Satellite Rendezvous”. *Journal of Aerospace Science*, Vol.27, 1960.
- [19] M.C.Eckstein, “Safe Rendezvous Approach to A Space by Impulsive Transfers and Continuous Thrust Arcs.” Proc. First European In Orbit Operations Technology Symposium, Nov. 1987, N88–19486.
- [20] F.Hechler, “Safe and Fuel Minimum Reference Trajectories for Closed Loop Controlled Approaches.” Proc. First European In Orbit Operations Technology Symposium, Nov. 1987, N88–19488.
- [21] 潘科炎,“飞船的双冲量最优交会”, 航天控制, 1991年第2期。

[22] 许论辉, 任萱, “四次冲量时间不固定的交会最优解”, 863高技术205-4会议论文, 承德, 1989年11月。

[23] 谢颖, 王旭东, “多冲量最优交会”, 航天控制, 1992年第1期。

[24] L.R.Gross, and J.E.Pruessing, “Optimal Multiple-Impulse Direct Ascent Fixed-Time Rendezvous.” AIAA Journal, Vol.12, No.7, July 1974.

[25] K.G.Eckel, “Optimal Impulsive Transfer With Time Constraint,” Acta Astronautica Vo.9, No.3, 1982.

[26] K.G.Eckel, and N.X.Vinh, “Optimal Switching Conditions for Minimum Fuel Fixed Time Transfer Between Non-Coplanar Elliptical Orbit.” Acta Astronautica, Vol.11, No.10 / 11, 1984.

[27] J.E.Pruessing, and J.H.Chiu, “Optimal Multiple-Impulse Time-Fixed Rendezvous Between Circular Orbit,” Journal of Guidance, Control, and Dynamics, Vol.9, No.1, Jan. -Feb. 1986.

[28] 谢颖, 黄文虎, 倪茂林, 王旭东, 白拜尔, “多冲量最优交会的动态规划方法”, 宇航学报, 1993年第2期。

[29] 谢颖, 王旭东, 倪茂林, “水平推力作用下共面椭圆轨道的最优转移”, 航天控制, 1993年第1期。

[30] 谢颖, 王旭东, 何<sup>1</sup>, “水平冲量作用下共面椭圆轨道上航天器的交会”, 控制工程, 1993, No. 3.

## Development of the Control Theories and Methods on Optimal Rendezvous in Space

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**Abstract** *The paper Surveys the Control theories and methods on optimal rendezvous in space at home and abroad, giving emphasis to the development of recent years at home, and introduces research results in the area.*

**Subject terms** *Rendezvous Docking Optimum rendezvous Control theory*

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